Journal of Economic & Financial Studies, 02(02), 41-44

Vol. 02, No. 02: April (2014)

Journal of Economic & Financial Studies

Open access available at http://journalofeconomics.org

X-Sigma-Rho and Market Efficiency

G. Glenn Baigent [a*]

[a] LIU – Post, College of Management, 720 Northern Blvd., Brookville, NY 11548
* Corresponding author's email: glenn.baigent@liu.edu

HIGHLIGHTS:

1. Focus on the incompleteness of Menchero and Davis (2011) work
2. X-sigma framework has been applied.
3. Focus has been given to the marginal contribution to return and risk simultaneously.
4. If securities are of same risk-return characteristics, the security with lower correlation to the portfolio will provide the higher improvement in efficiency.

Article History

Received: 10-03-2014
Accepted: 14-04-2014
Available online: 26-04-2014

Keywords:
Market efficiency;
Performance measure;
X-sigma rho.

JEL Classification:
G20.

ABSTRACT

Menchero and Davis (2011) define X-Sigma-Rho as a risk metric that shows the marginal contribution to risk when a security is added to a portfolio of other securities. While insightful regarding risk, their work is incomplete because it does not consider the marginal contribution to return. This paper completes their analysis by including marginal contribution to return. In equilibrium the result is the capital market line and a measure similar to Jensen's alpha that can be used to measure performance.

DOI: http://dx.doi.org/10.18533/jefs.v2i02.133

1.0 Introduction

A fundamental assertion of financial economic theory is that individuals will accept risk only if compensated through return. Moreover, the return must be increasing in the measure of risk implying that individuals have an aversion to risk and will judge expected return relative to underlying risk. Menchero and Davis (2011) define *X-Sigma-Rho* as a risk metric that shows the marginal contribution to risk when a security is added to a portfolio of assets. In this analysis the "*X-Sigma-Rho"* result is extended by adding marginal contribution to return.

If both marginal risk and marginal return are considered under the equilibrium condition that all assets have the same reward to risk ratio, say Sharpe's ratio, the capital/security market line (CML/SML) easily obtains from the *X-Sigma-Rho* framework. Thus, the *Sigma-Rho* part of Menchero and Davis' metric is not surprisingly a determinant of expected return using either total or systematic risk.

When the CML is expressed in *Sigma-Rho* terms the impact of diversification on relevant risk is evident. If there is no diversification effect (the security's correlation with the market portfolio is +1.0) the CML provides an estimate of expected return based on *total* risk. For all other cases where at least some diversification exists (the security is not perfectly positively correlated with the market portfolio) the expected return is from the security market line or capital asset pricing model [Sharpe, 1964] and is dependent upon systematic risk. In this way the CML is a special case of the SML when *Rho* is equal to +1.0.

In this framework the expected rate of return is a function of a security's standard deviation relative to the standard deviation of the market (risk factor), but this risk factor is scaled by the correlation, rho, and can become a maximum of σ_i/σ_M or a minimum of $(-)\sigma_i/\sigma_M$. Regardless of the value of rho, a *normal* rate of return can be determined so that the equilibrium or risk-based rate of return can be compared to the observed return to identify mis-priced assets, a measure that is a form of Jensen's Alpha.

The *Sigma-Rho* approach is compelling because it highlights the impact of correlation on portfolio formation. Strategies such as creating a zero-beta portfolio, a market-neutral portfolio, or the use of precious metals and other commodities have at their roots the correlation of securities; portfolios formed on this basis should tend towards the efficient frontier. The end result is, for two assets with the same risk-return characteristics, the security with the lower correlation to the existing portfolio will contribute the greatest efficiency. This can only be achieved if both return and risk are considered.

2.0 X-sigma-rho revisited

Menchero and Davis (2011) and Davis and Menchero (2012) study the attribution of risk in the context of standard deviation and correlation. For completeness, *X-Sigma-Rho* is reviewed in this section using the same notation.

Define a portfolio's return as R in equation (1) where g_m and x_m are the return and proportional weight (or exposure) for each security m.

$$R = \sum_m x_m g_m \tag{1}$$

Equation (1) is a weighted-average of the component returns constrained by $\sum_m x_m = 1$. From (1), it follows that the variance or volatility of returns is

$$\sigma^2(R) = \sum_m x_m Cov(g_m, R) \tag{2a}$$

or,

$$\sigma^2(R) = \sum_{n,m} x_n x_m Cov(g_m, g_n) \tag{2b}$$

The main thrust of Menchero and Davis' (2011) analysis is to derive a risk metric they refer to as *X-Sigma-Rho.* In (3), $\sigma(g_m)$ is the standard deviation or volatility of security m and $\rho(g_m, R)$ is the correlation of security m with the remainder of the portfolio, R. The volatility of the portfolio is,

$$\sigma(R) = \sum_m x_m \sigma(g_m)\rho(g_m, R) \tag{3}$$

Menchero and Davis show that differentiating (2b) with respect to x_m gives the marginal contribution to risk (4).

$$MCR_m = \sigma(g_m)\rho(g_m, R) \tag{4}$$

Equation (4) describes the marginal contribution to risk for a security as a function of its individual volatility scaled by its correlation to the remainder of the portfolio's securities. This is a very compelling framework because of the boundaries of ρ. That is, if $\rho = +1$ the full risk of security m applies as there is no diversification effect since portfolio risk is identical to weighted-average risk. However, if $\rho < +1$, there is a diversification effect where portfolio risk is less than weighted-average risk and the difference between the two is dependent upon the value of ρ. Their analysis is even more intuitive with the observation that the marginal contribution to portfolio risk can be shown as monotone decreasing in ρ where the lower the correlation, the lower the standard deviation, *ceteris paribus*.

3.0 Sigma-rho-equilibrium

Menchero and Davis' *X-Sigma-Rho* equation has a broader application when considered in the context of marginal return (MR). Differentiating (1) with respect to x_m gives the marginal contribution to return as

$$\frac{\partial R}{\partial x_m} \Rightarrow MR_m = g_m \tag{5}$$

The marginal return of security m is the contribution to a portfolio's return but it should be examined in the context of adjustment for risk. To make this adjustment, the commonly used Sharpe measure is employed as a ratio of the marginal contribution of return to the marginal contribution to risk and comparing it to a "market portfolio", M, or some other efficient benchmark. A long-only portfolio manager should want to add (remove) security m if its marginal risk-adjusted contribution to return is positive (negative). It might also be the case that a portfolio manager does not include a security with a positive Sharpe ratio because it does not contribute enough return to compensate for its risk. This issue was addressed by Elton, Gruber, and Padberg (1978). [In this analysis the typical and sometimes controversial assumptions for the market portfolio apply.] The Sharpe ratio for security m is,

$$S_m = \frac{g_m - r_F}{\sigma(g_m)\rho(g_m, M)} \tag{6}$$

In equilibrium the expectation is that all assets will have the same reward to risk ratio so that (7a) obtains, where M is the market portfolio.

$$\begin{gathered} S_m = S_M \\ \frac{g_m - r_F}{\sigma(g_m)\rho(g_m, M)} = \frac{g_M - r_F}{\sigma(M)} \end{gathered} \tag{7a}$$

Simplifying (7a) gives

$$g_m = r_F + \left(\frac{\sigma(g_m)\rho(g_m, M)}{\sigma(M)}\right)[g_M - r_F] \tag{7b}$$

Equation (7b) is Sharpe's (1964) capital asset pricing model (CAPM) or the security market line, and when $\rho = +1$, (7b) is equivalent to the capital market line, the difference being the consideration of total risk versus systematic risk. (Equation 7b can also be easily derived under no-arbitrage conditions as shown in the Appendix.)

For total risk adjustment let $\rho = +1.0$ and for systematic risk consideration let $\rho \neq +1.0$. Therefore, *X-Sigma-Rho* leads to a restatement of Jensen's Alpha (Jensen, 1968) as shown in (8). This is denoted *Sigma-Rho-Alpha*.

$$\Sigma\text{PA} = g_m - \left\{ r_F + \left(\frac{\sigma(g_m)\rho(g_m, M)}{\sigma(M)}\right)[g_M - r_F] \right\} \tag{8}$$

Equation (8) defines "alpha" as a function of the risk free rate of return, individual standard deviations, the efficiency of the benchmark return (g_M), and the correlation of the security with the benchmark. Ceteris paribus, as ρ decreases, alpha increases because the risk premium decreases. This level of efficiency should be the objective of the analyst.

Using the methodology described here, there should be a tendency towards efficient markets with correctly priced assets as (8) describes arbitrage opportunities. In well functioning liquid markets, mis-priced assets should be restored to equilibrium. However, this process will not take place without the presence of return so that it is necessary to extend Mencharo and Davis' (2011) analysis.

4.0 Conclusion and managerial implication

The risk analysis conducted by Menchero and Davis adds to the literature as *X-Sigma-Rho* but additionally provides a framework for deriving the capital market line and the capital asset pricing model under equilibrium conditions. If we require that all assets have the same marginal contribution to return after adjusting for marginal contribution to risk, the CAPM easily obtains. Analysts can improve the efficiency of portfolios by jointly considering the marginal rate of return and the marginal contribution of risk. Given two securities with the same risk-return characteristics, the security with lower correlation to the portfolio will provide the higher improvement in efficiency.

References

Amenc, N., and L. Martellini, 2012. Forget about alpha. The Journal of Portfolio Management, 38(04): 04 05.

Baigent, G., 2005. Six simple proofs for intro to finance courses. Advances in Financial Education, 03: 5-47.

Davis, B, and J. Menchero., 2012. The alpha and beta of risk attribution. The Journal of Portfolio Management, 38 (02): 99-107. http://dx.doi.org/10.3905/jpm.2012.38.2.099

Elton, E. M. Gruber, and M. Padberg., 1978. Simple criteria for portfolio selection: tracing out the efficient frontier. Journal of Finance, 33(01): 296-302. http://dx.doi.org/10.1111/j.1540-6261.1978.tb03407.x

Jensen, M. C., 1968. The performance of mutual funds in the period 1945-1964. Journal of Finance, 23(02): 389-416. http://dx.doi.org/10.1111/j.1540-6261.1968.tb00815.x

Menchero, J., and B. Davis., 2011. Risk contribution is exposure time's volatility time's correlation: decomposing risk using the x-Sigma-rho formula. The Journal of Portfolio Management, 37(02): 97-106. http://dx.doi.org/10.3905/jpm.2011.37.2.097

Sharpe, W. F., 1968. Capital asset prices: a theory of market equilibrium under conditions of risk. Journal of Finance, 19(03): 425-442. http://dx.doi.org/10.1111/j.1540-6261.1964.tb02865.x, http://dx.doi.org/10.2307/2977928

Appendix

X-Sigma Rho and the SML/CML

Form a portfolio with a as the amount allocated to security m and $(1-a)$ allocated to the risk free asset. Because of the zero correlation between g_m and r_F the combination is linear and can be computed as the weighted-average volatility. Equating the volatility of this combination or portfolio to volatility of the market portfolio gives the portfolio weights shown in A.1.

$$a\sigma(g_m)\rho + (1-a)\sigma_F = \sigma(M)$$

$$a = \frac{\sigma(M)}{\sigma(g_m)\rho} \quad \text{and} \quad (1-a) = 1 - \frac{\sigma(M)}{\sigma(g_m)\rho} \qquad \text{(A.1)}$$

Since this portfolio has the same risk as the market portfolio it should have the same rate of return. Equating the returns yields (A.2).

$$ag_m + (1-a)r_F = g_M$$

$$\left(\frac{\sigma(M)}{\sigma(g_m)\rho}\right)g_m + \left(1 - \frac{\sigma(M)}{\sigma(g_m)\rho}\right)r_F = g_M \qquad \text{(A.2)}$$

Simplify A.2 by multiplying through by $\sigma(g_m)\rho/\sigma(M)$ and the SML/CML obtains. A similar proof is shown in Baigent (2005).