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# Valuing guaranteed bank debt: Role of strength and size of the bank and the guarantor\*

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ABSTRACT

A contingent claims model of the value of sovereign guarantees of bank debt shows that the value decreases with the bank's own creditworthiness and increases with that of the sovereign as well as with bank and sovereign size. Using cross-sectional data for 188 large banks world-wide from 2007 to 2013, empirical results are consistent with the model's implications, suggesting that the implicit support for a bank is higher when the bank is larger, when the bank is weaker, and when the country in which the bank is headquartered is larger or more creditworthy. While bank-specific factors matter as well as those related to the sovereign of the country where the bank is located, the effect on the value of sovereign guarantees of changes in bank strength dominate those in sovereign strength.

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#### 1.0 **Motivation**

All debt is risky. This principle is generally acknowledged in the modeling of the debt of private entities, including banks, and of debt issued by sovereigns in emerging economies. It is more likely to be downplayed and has even been overlooked until recently in the case of sovereign debt or debt guarantees in more advanced economies, where however the public provision of the function of guarantor-of-last resort was a key element of the policy response in many OECD countries (Schich, 2013). Already before these events, Gray, Merton, and Bodie (2007) contended, the ability of the sovereign to pay off its debt or to make good on guarantees on private sector debt can play a major role in the valuation and risk assessment of private sector debt instruments.

More recently, the role of the strength of the sovereign for the value of the debt of domestic banks has come into even sharper focus; in fact, a strong sovereign tends to provide a more valuable guarantee than a weak sovereign for bank debt of a given intrinsic quality (Estrella and Schich, 2011). The present paper extends this conceptual and empirical analysis by focusing more sharply on the roles of the strength of the bank and the sovereigns, respectively, in explaining the value of guaranteed bank debt.

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Section 2.02 presents technical details of the basic model and Section 2.03 provides numerical valuation results and sensitivity analysis to illustrate the relationships between bank strength and sovereign strength on the one hand and the increase in value of guaranteed bank debt on the other. Section 2.04 discusses the role of the strength as well as size of the bank compared to that of the sovereign for the value of a sovereign bank debt guarantee in more detail. The paper also reports the results of an empirical analysis. This conceptual framework developed here is in principle valid both for the case of explicit as well as implicit guarantees. In particular, the conceptual implications are the same as long as the latter is unanimously assumed to exist. Section 3.0 provides the results of an empirical analysis and Section 4.0 concludes.

# 2.0 The conceptual framework

## 2.01 Motivation for the model

The model of this paper is adapted from Estrella and Schich (2011). It is an extension of the Merton (1974) framework for the valuation of risky debt of a single issuer, which has been applied and extended in various ways in the finance literature. The framework is particularly helpful in the case of banks, which tend to have thin capital margins (high leverage) and thus a propensity for large proportional equity losses in response to changes in asset value. The extension here involves the modeling of two separate sources of risk that may or may not be correlated.

We are concerned in this article with the valuation and risk assessment of bank debt guaranteed by a sovereign. Thus, our model is designed to include two sources of risk, as well as the correlation between them. Other earlier extensions of Merton's (1974) approach to two sources of risk include Johnson and Stulz (1987), who calculate the value of a debt guarantee under the assumption that the guarantor has no debt outstanding and Lai (1992), who proposes an approximate closed-form expression for the value of the guarantee when the guarantor has no debt numerical results in the case in which the guarantor has nonzero debt. Our approach is a general extension of the Merton (1974) model to two sources of risk using a bivariate counterpart to Merton's univariate stochastic distribution. The resulting form of the model does not lend itself to closed-form expressions for the value of guaranteed debt, but we can readily generate numerical values from a calibrated version of the model.

# 2.02 Basic model to value guarantees from a risky guarantor

#### 2.2.1 Assumptions regarding payoff structure

Our model includes two agents operating in a single-country setting, a bank and the sovereign, and both issue risky debt. For valuation purposes, we consider the debt of each agent on a standalone basis, but our main focus is on the debt of the bank under the assumption that the sovereign provides a financial guarantee. We look at the bank's debt from the perspective of an investor in the bank, with or without a guarantee. As to the guarantor, our assumption here is that the sovereign's own liabilities are senior to its guarantee of bank debt. Thus, the guarantee does not directly affect the value of the sovereign's direct debt issuance. Other assumptions are possible and could induce changes in the valuation of sovereign debt, but we leave those alternatives for future research.

We compute the value of debt for each entity using contingent claims pricing. For the bank, we consider both the stand-alone value and the value if there is a guarantee from the sovereign. For simplicity, we consider a one-period model in which debt is issued at time 0 and matures at time 1. The assets of the bank and the sovereign evolve as a bivariate continuous-time Wiener process, which is a direct extension of Merton (1974). The distribution of time 1 assets is bivariate lognormal, which leads to straightforward numerical calculations, though not in general to closed-form solutions.

For stand-alone bank debt, the payoff at time 1 is

$$R_{B} = \begin{cases} L_{B} & \text{if } A_{B} \ge L_{B} \\ A_{B} & \text{if } A_{B} < L_{B} \end{cases},$$
(01)

where  $A_{B}$  and  $L_{B}$  are the assets and liabilities (face value of debt) of the bank.

The payoff may also be expressed as

$$R_{B} = L_{B} - \begin{cases} 0 & \text{if } A_{B} \ge L_{B} \\ L_{B} - A_{B} & \text{if } A_{B} < L_{B} \end{cases}$$
(02)

where the second term is equivalent to a put option on the bank's assets with strike price equal to its debt.

In some earlier analyses of sovereign debt guarantees, the sovereign is assumed to be risk free. Here we follow, for example, the theoretical discussion in Gray, Merton, and Bodie (2007) and assume that the sovereign's assets are also subject to risk and that its debt is therefore risky. By analogy to the bank case, the payoff of sovereign debt is

$$R_{S} = \begin{cases} L_{S} & \text{if } A_{S} \ge L_{S} \\ A_{S} & \text{if } A_{S} < L_{S} \end{cases}$$
(03)

and it also has a put option interpretation corresponding to (2). Gray, Merton, and Bodie (2007) suggest that we can think of sovereign assets as foreign reserves, net fiscal assets, and other public assets, and that sovereign liabilities may include foreign-currency debt, local-currency debt, base money, and guarantees.

In addition to its direct liabilities, the sovereign may also provide a guarantee on the debt of the bank. In this case, sovereign assets at the end of the period, if they exceed sovereign liabilities, may be used to cover any shortfall of bank assets in covering the face value of bank debt. As noted earlier, we assume that the sovereign's own debt is senior to the bank guarantee. Thus, the payoff for guaranteed bank debt is as summarized in Table 01.

**Table 01:** Conditional payoffs for bank debt guaranteed by the sovereign

Case	Payoff $R_G$ is	If
1	$L_{\scriptscriptstyle B}$	$A_B \ge L_B$
2	$L_{\scriptscriptstyle B}$	$A_B < L_B$ and $A_S \ge L_S + L_B - A_B$
3	$A_B + A_S - L_S$	$A_B < L_B$ and $L_S \le A_S < L_S + L_B - A_B$
4	$A_{\scriptscriptstyle B}$	$A_B < L_B$ and $A_S < L_S$

We price the bank and sovereign debt using the extended Merton (1974) methodology, making the following assumptions.

- 1. Both the bank and the sovereign have debt outstanding. (In contrast to earlier work in which the sovereign is frequently assumed to have no debt.)
- 2. Sovereign debt is senior to the bank guarantee.
- 3. Bank and sovereign assets follow a bivariate Wiener process and their values are jointly lognormal. (In contrast to some earlier work that assumes the sovereign is risk-free.)
- 4. A closed-form solution for guaranteed debt is not feasible since it involves the sum of lognormal variables. We therefore use numerical integration of the exact bivariate lognormal density.
- 5. We apply risk-neutral valuation as in Merton (1974).

#### 2.2.2 Risk-neutral valuations of different claims

For risk-neutral valuation purposes, we use the bivariate Wiener process

$$\begin{bmatrix} dx/x \\ dy/y \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix} dt + \Sigma dW, \qquad (04)$$

where x and y correspond to bank and sovereign assets, respectively, dW is two-dimensional standard Brownian motion, and

$$\Sigma\Sigma' = \begin{bmatrix} \sigma_B^2 & \rho\sigma_B\sigma_S \\ \rho\sigma_B\sigma_S & \sigma_S^2 \end{bmatrix}.$$
 (05)

For example,  $\Sigma$  could be the Cholesky decomposition of  $\Sigma\Sigma'$  in (5), that is,

$$\Sigma = \begin{bmatrix} \sigma_B & 0\\ \rho \sigma_S & \sqrt{1 - \rho^2} \sigma_S \end{bmatrix}.$$
 (06)

The joint lognormal density function for asset values at time  $\tau$  is then:

$$f(x,y) = \frac{1}{2\pi\sigma_B\sigma_S\tau\sqrt{1-\rho^2}} \frac{1}{xy} \exp\left(-\frac{u(x)^2 + v(y)^2 - 2\rho u(x)v(y)}{2(1-\rho^2)}\right),$$
(07)

where  $u(x) = \left(\log(x) - \left(r - \frac{1}{2}\sigma_B^2\right)\tau\right) / \sigma_B \sqrt{\tau}$  and  $v(y) = \left(\log(y) - \left(r - \frac{1}{2}\sigma_S^2\right)\tau\right) / \sigma_S \sqrt{\tau}$ .

The payoff at time  $\tau$  of stand-alone bank debt is given by (1) and its value is obtained from the marginal distribution  $f_B(x) = \int_0^\infty f(x, y) dy$  implied by (7):

$$V_{B} = e^{-r} \left[ \int_{L_{B}}^{\infty} L_{B} f_{B}(x) dx + \int_{0}^{L_{B}} x f_{B}(x) dx \right].$$
(08)

The payoff at time  $\tau$  of sovereign debt is given by (3) and its value is similarly obtained from the marginal distribution  $f_s(y) = \int_0^\infty f(x, y) dx$  implied by (7):

$$V_{S} = e^{-r} \left[ \int_{L_{S}}^{\infty} L_{S} f_{S}(y) dy + \int_{0}^{L_{S}} y f_{S}(y) dy \right].$$
(09)

For bank debt guaranteed by the sovereign, the payoff at time  $\tau$  is given by Table 01 and its value is obtained from the bivariate distribution (7) as

$$V_{G} = e^{-r} \left[ \int_{0}^{\infty} \int_{L_{B}}^{\infty} L_{B} f(x, y) dx dy + \int_{L_{B}+L_{S}-x}^{\infty} \int_{0}^{L_{B}} L_{B} f(x, y) dx dy + \int_{L_{S}}^{L_{B}+L_{S}-x} \int_{0}^{L_{B}} (x + y - L_{S}) f(x, y) dx dy + \int_{0}^{L_{S}} \int_{0}^{L_{B}} x f(x, y) dx dy \right].$$
(010)

Although the Merton (1974) model and its extensions are widely used in asset pricing theory, there is evidence that financial asset returns tend to follow probability distributions that exhibit greater leptokurtosis than the normal distribution. That is, large deviations from the mean are more common than with the normal distribution, leading to a condition popularly known as "fat tails." For example, Fergusson and Platen (2006) find that a Student's t distribution with 4 degrees of freedom, which has fatter tails than the normal, fits data for stock index returns fairly accurately.

#### 2.03 Sensitivity of guaranteed bank debt to changes in variable or parameter values

#### 2.3.1 Approach taken to analyze sensitivity of debt value

To analyze the characteristics of the proposed model and examine its implications for debt guarantees, we provide numerical results based on the following calibration of model parameters. Parameters in the illustrations assume the following base case values unless otherwise noted. We take the time period loosely to represent one year and the riskless interest rate to be 3 per cent on a per-period compounded basis. The value of bank assets is 100 at time 0 and the face value of bank liabilities, payable at time 1, is 95. Although these values seem to indicate that the bank has formally a 5 per cent capital ratio, when discounting and risk are taken into account, the true value of the capital ratio is closer to 12 per cent.

We take time 0 sovereign assets to be 200 and the face value of sovereign liabilities to be 180. The volatility of asset returns is 0.3 for both the bank and the sovereign. For the correlation of bank and sovereign asset returns, which we denote as "rho", we look at two cases, zero and 0.8. A higher correlation tends to undercut the value of the sovereign's debt guarantee, since the sovereign's financial position would tend to be unfavorable when a potential bank default occurs.

#### 2.3.2 Sensitivity of bank debt value to parameters describing the bank

Figure 01 shows the value of bank debt as a function of the initial value of bank assets, as the latter varies from 90 to 200, with all other parameter values fixed at the base case levels. The vertical dotted line indicates the base case face value of liabilities. Debt values are presented under four different sets of assumptions. An upper bound is given by the present value (PV) of face value, which corresponds to the assumption that the debt is riskless (dashed line). This value is independent of initial bank assets. A lower bound is provided by the standalone value of the debt, which assumes that there is no guarantee from the sovereign or any other source (dotted line).

When there is a guarantee from the sovereign, the value of the guarantee depends on the correlation between bank and sovereign assets, as noted earlier. When the correlation is high (rho=0.8), the uplift in bank debt value provided by the guarantee is very modest, as indicated by a comparison of the standalone value and the guaranteed value (dash-dot line). A much more substantial effect on the value of the guarantee occurs when the correlation is zero, in which case the financial state of the guarantor is independent from that of the bank and the sovereign may be in good shape to cover the payment of bank debt when the bank is in a default state.

Note that, in all but the riskless case, the sensitivity of the value of debt to bank asset value (the bank asset "delta") is substantially larger for lower asset values. This sensitivity approaches zero as the initial value of bank assets grows well beyond the base case value of 100. This nonlinearity means that the value of debt becomes more volatile as the condition of the bank deteriorates.



Notes: For a given face value of bank debt of 95 at time t=1, the figure shows the value of such debt as a function of the level of bank assets at time t=0. Initial bank assets vary from 90 to 200. Sovereign assets and liabilities are fixed at 200 and 180, respectively. "rho=0" and "rho=0.8" show the value of guaranteed bank debt assuming correlation of zero and 0.8, respectively, between bank and sovereign asset returns. "Standalone value" abstracts from the existence of a guarantee.

#### 2.3.3 Sensitivity of bank debt value to parameters describing the sovereign

We next perform a similar exercise by letting the initial value of sovereign assets vary from 50 to 600, with the value of bank assets and all other parameters at their base case levels. Results appear in Figure 02. Once again we show the upper and lower bounds given by the present value of face value and the standalone value, respectively, which in this case are both independent of sovereign asset value and appear as horizontal lines. The vertical dotted line represents the base case face value of sovereign liabilities.

As in the case of the sensitivity to bank assets, sensitivity to sovereign assets grows nonlinearly from the lower to the upper bound. However, the nonlinearity in this case is different in that bank debt value is less sensitive when sovereign assets are low as well as high. Sensitivity is much higher in an intermediate range of sovereign asset values. In practical terms, this means that changes in the financial condition of the sovereign have little import for bank debt both when the sovereign's financial condition is very strong (the guarantee makes bank debt close to riskless) and very weak (the guarantee is close to worthless). In the intermediate range, it is clear again that the uplift provided by the sovereign is less if the assets of the two agents are more highly correlated.



Notes: For a given face value of bank debt of 95 at time t=1, the figure shows the value of such debt as a function of the level of sovereign assets at time t=0. Initial sovereign assets vary from 50 to 600. Bank assets and liabilities are fixed at 100 and 95, respectively. "rho=0" and "rho=0.8" show the value of guaranteed bank debt assuming correlation of zero and 0.8, respectively, between bank and sovereign asset returns. "Standalone value" abstracts from the existence of a guarantee.

#### 2.3.4 Sensitivity of bank debt value to parameters describing the bank and the sovereign

Figure 03 shows the uplift in the value of bank debt, that is, the difference between the guaranteed and standalone values as a function of both bank and sovereign assets (for a given amount of liabilities). The correlation is zero

in the figure. The shape of the function is qualitatively the same if the correlation is higher (for example, 0.8), but the numerical values of the uplift are lower.

Figures 01 and 02 show that the value of guaranteed bank debt is positively related to the value of both bank and sovereign assets. Similarly, the uplift is also positively related to the value of sovereign assets. However, Figure 03 shows that the uplift is negatively related to bank assets. Intuitively, a bank in worse financial condition stands to gain more from a guarantee than one in better shape.



Notes: For a given face value of bank debt of 95 at time t=1, the figure shows the uplift in the value of such debt as a function of the level of bank assets at time t=0. Initial bank assets vary from 90 to 200. Sovereign assets and liabilities are fixed at 200 and 180, respectively. "rho=0" and "rho=0.8" show the uplift in the value of guaranteed bank debt assuming correlation of zero and 0.8, respectively, between bank and sovereign asset returns.

The sensitivities of bank debt to other parameters of the model are largely straightforward. For instance, the value of guaranteed bank debt responds positively to increases in the scale of the sovereign if the financial condition of the sovereign (ratio of assets to liabilities) is held fixed. That is, a large country can take better care of a small bank than when the sizes are reversed.

Higher volatility of sovereign assets means that the guarantee is riskier, decreasing the value of bank debt correspondingly. Higher volatility of bank assets also has in general a negative effect on the value of bank debt, but non-monotonic relationships have been observed by, for example, Stulz and Johnson (1985) and Lai (1992) when the asset return correlation is close to -1 or when the ratio of bank to sovereign volatility is either very large or very small. We find limited evidence of these results if we start from our base case calibration.

The effect of changes in asset correlation is generally as indicated in Figures 01-02 and discussed earlier, even if the correlation is negative. Lower correlation provides a kind of diversification effect and enhances the value of the guarantee and the value of bank debt.

We conclude this section with a brief note about the tails of the theoretical distributions. Some of the literature on asset returns suggests that empirical distributions tend to be leptokurtic of fat-tailed. We have examined this question elsewhere by substituting a bivariate log-t distribution with 4 degrees of freedom in both dimensions for the normal distribution. Results are numerically a bit different, as would be expected, but results using the t distribution are qualitatively the same as those using the normal distribution. Thus, for our present purposes in illustrating theoretical relationships, leptokurtosis does not play a material role.

# 2.04 Uplift from the guarantee on the value of bank debt

Figure 04 below shows the value of the uplift provided to risky bank debt by a sovereign guarantee when the sovereign itself can be risky. Bank and sovereign liabilities are assumed to be equal to 95 and 180, respectively, and the scenarios differ depending on the assumptions regarding the variation in bank and sovereign assets. Four scenarios are considered:

 Sov 200: This scenario assumes a constant level of sovereign assets equal to 200 ("weak sovereign"), and the line shows how the value of the uplift decreases as bank assets increase from 100 to about 200. In fact, the value of the uplift converges towards zero as the bank strength, as measured by the difference between  $A_{R}$  and  $L_{R}$ , increases.

- *Sov 300*: This scenario assumes a constant level of sovereign assets equal to 300 ("strong sovereign"), and the straight line shows how the value of the uplift decreases as bank assets increase from 100 to 200. It shows that the value of the guarantee again converges towards zero. Compared to the first scenario, that is *Sov 200* with a weaker sovereign, the decrease in the value of the uplift, as bank assets increase, starts from a higher level and occurs at a faster pace.
- *Sov low path*: This scenario assumes a constant level of bank assets equal to 100, while sovereign assets increase from 200 to 400. The uplift provided by the guarantee increases reflecting the increase in the sovereign strength.
- Sov high path: As in the previous scenario, bank assets are fixed at 100, but this time sovereign assets increase from 300 to 600. Given that the sovereign is already quite strong initially, further increases in sovereign assets provide more limited contribution to the uplift of bank value debt compared to the previous scenario. The slope of the line is flatter compared to the previous scenario.



Notes: For a given face value of bank debt of 95 at time t=1, the figure shows the value of such debt as a function of changes in the level of both bank and sovereign assets at time t=0. Initial bank assets vary from 90 to 200. Sovereign assets are fixed at 200 ("Sov 200") or 300 ("Sov 300"), respectively. Initial sovereign assets vary from 200 to 400 ("Sov low path") and from 300 to 600 ("Sov high path"), respectively. Bank and sovereign asset returns are uncorrelated.

The computations also illustrate that the value of a guarantees can be the same for different combinations of bank and sovereign strength, e.g. where *Sov 300* and *Sov low path* are intersecting. Another way to look at such combinations is by calculating iso-uplift contours, that is, curves over which the uplift is constant. In Figure 05, bank and sovereign liabilities are set at 95 and 190, respectively, as assets are allowed to vary. The figure shows that the uplift is more sensitive to differences in bank assets than to differences in sovereign assets. Bank risk seems more important from this perspective.



Notes: Iso-uplift curves show combinations of the levels of initial bank and sovereign assets at time t=0 for which the uplift in the value of bank debt are identical, for a given face value of bank debt of 95 at time t=1. Initial bank and sovereign assets vary from 100 to 150 and from 200 to 300, respectively. Bank and sovereign asset returns are uncorrelated.

## 2.05 Variations in the relative size and strength of the bank and the sovereign

An alternative way of illustrating the relative roles for the value of bank debt guarantees of bank versus sovereign strength is shown in Figures 06 and 07. These figures display the results of computations based on the model extensions contained in the Appendix, where the size of the bank is kept constant while guarantor size or solvency are allowed to vary.

Figure 06 illustrates that the uplift is more sensitive to changes in bank assets than sovereign assets. Sensitivity of uplift to bank asset changes is the solid line, to sovereign asset changes is dots and dashes, and the relative sensitivity adjusting for sign is the dashed line. The relative sensitivity ranges between close to zero and a maximum of about 0.8 and hence is always below one. This observation implies that, for any given uplift, the sensitivity to changes in sovereign assets is always less than the sensitivity to changes in bank assets, or alternatively, that bank assets matter more for the uplift.

In Figure 07, the size of the sovereign is allowed to vary, but keeping its solvency ratio constant. Thus, this figure illustrates the sensitivity of uplift to bank and sovereign asset changes when only the relative size of the sovereign changes. The results are qualitatively similar to those in Figure 06, where both sovereign size and solvency change, but the ratio decreases toward zero more gradually in Figure 07 with solvency held constant.



Notes: The figure shows the local sensitivities of the uplift function to changes in bank or sovereign assets as well as their ratio, which is the relative sensitivity for a constant value of uplift  $(dA_B/dA_S|_{U=U_0})$ . Bank and sovereign liabilities are each fixed at 95 and bank assets at 100. Sovereign assets vary between 100 and 400 and solvency ratio increases with sovereign assets. Bank and sovereign asset returns are uncorrelated.

Figure 07: Relative sensitivities of the uplift in value of guaranteed bank debt as sovereign becomes larger



Notes: The figure shows the local sensitivities of the uplift function to changes in bank or sovereign assets as well as their ratio, which is the relative sensitivity for a constant value of uplift ( $dA_B/dA_S|_{U=U_0}$ ). Bank liabilities are fixed at 95 and bank assets at 100. Sovereign liabilities are 95 per cent of sovereign assets and, thus, the solvency ratio remains constant as sovereign assets vary between 100 and 400. Bank and sovereign asset returns are uncorrelated.

Earlier, Figures 01 and 02 showed the value of guaranteed bank debt as a function of bank and sovereign assets, while Figure 03 showed the uplift as a function of bank assets, with liabilities fixed in all these cases. Next, we consider the effect of changes in size of bank or sovereign on the uplift, keeping the ratio of assets to liabilities but not their absolute level constant. The effect of changes in bank or sovereign size is assessed here by varying each entities' assets and liabilities, keeping the ratio between the two variables constant.

Figure 08 shows the following. When only sovereign assets change but sovereign liabilities stay constant, bank debt uplift increases with sovereign assets as the sovereign gets "stronger" in the sense of being more solvent (if assessed on its own). When sovereign assets and liabilities change proportionately, the sovereign size effect on the value of bank debt is less because the sovereigns' solvency ratio stays the same. It has the same sign (i.e. positive) as when sovereign assets alone change, although the increase in uplift with increases in sovereign size increasing is very small, as shown by the dotted line.



Notes: Bank liabilities are fixed at 95 and bank assets at 100. Sovereign assets vary from 200 to 400, with either liabilities fixed at 180 (solid line) or increasing with assets at a constant ratio of 180/200 or 90%, so that the sovereign solvency ratio remains constant (dotted line). Bank and sovereign asset returns are uncorrelated.

Figure 09 performs a similar exercise with regard to bank size. It shows that the effect of changes in bank size (assets & liabilities in proportion) is more dramatic in this case because it changes sign from negative to positive. The solid line shows that uplift decreases with bank assets (for given bank liabilities) because the bank becomes more solvent and thus receives or "needs" less help from the unchanging sovereign. When bank assets and liabilities grow proportionally, however, the effect on uplift of an increase in bank size is positive. The intuition is that as liabilities are larger (in proportion to assets), the bank can use more help from the sovereign to insure the debt, which is larger in absolute terms. Since the bank solvency ratio is constant, the capacity of the bank to self-insure is not increased.



Notes: Sovereign liabilities are fixed at 180 and sovereign assets at 200. Bank assets vary from 100 to 200, with either liabilities fixed at 95 (solid line) or increasing with assets at a constant ratio of 95/100 or 95%, so that the bank solvency ratio remains constant (dotted line). Bank and sovereign asset returns are uncorrelated.

## 3.0 Empirical analysis

# 3.01 Application to implicit guarantees for bank debt

#### 3.1.1 Implicit versus explicit guarantees

The model described in the previous section is applied here to a measure of the value of implicit guarantees for the debt of large banks. Earlier examples of such empirical studies include Estrella and Schich (2011), Schich (2013) and Cariboni et al. (2014). The model developed here implies that the uplift in the value of bank debt due to a sovereign guarantee is a positive function of the strength and size of the sovereign providing the guarantee, a positive function of the size of the bank and a negative function of the strength of the bank benefitting from the guarantee. This implication applies regardless of whether the guarantee is explicit or implicit, as long as the latter is common market perception; that is, as long as market participants unanimously assume a guarantee to exist.

#### 3.02 The data

#### 3.2.1 Dependent variable: Credit rating uplifts due to assumed external support

To proxy the extent of implicit guarantees, we use a measure of the difference between the standalone credit profile and the issuer credit rating in the case of the 188 large worldwide banks rated by the credit rating agency Moody's.

For some time now, credit rating agencies have rated banks by explicitly factoring in an estimate of the external support that the bank under consideration receives, either from its parent or from public authorities. In fact, rating agencies provide two types of ratings for a bank. First, an "issuer credit rating" (ICR) that factors in the possibility and likelihood of external support that the bank under consideration receives from its parent or public authorities, when needed. Second, an "intrinsic strength" or "stand-alone" rating (SACR) that abstracts from such support. The difference between the two types of ratings in the case of each bank is referred to here as UPLIFT (*i.e.* ICR minus SACR).

A positive UPLIFT reflects the existence of explicit or perceived implicit support from either the parent or the government, or other favorable factors such as access to central bank liquidity and emergency liquidity support. Whatever the specific form that the external support might take, it effectively facilitates the issuer's servicing of its debt, and is thus functionally equivalent to an implicit guarantee for that debt.

By far the most important element is the assumed support from the government and, therefore, this difference is now commonly used as an empirical measure of the extent of explicit or implicit support from the government (see e.g. CGFS, 2011; Packer and Tarashev, 2011; Estrella and Schich, 2011; Ueda and Weder di Mauro, 2012).

#### 3.2.2 Explanatory variables

The variable UPLIFT is regressed on three key dependent variables: (1) the stand-alone credit profile of the bank, SACR, a measure of the banks' intrinsic strength, (2) the issuer credit rating of the respective sovereign, SCR; a measure of the strength of the sovereign, and (3) the logarithm of the gross domestic product of the sovereign where the bank is headquartered, SIZE, a measure of the size of the sovereign.

Our data consists of cross-sections of observations for between 185 and 188 large banks from 23 countries for seven years from 2007 and 2013, with sample size varying by year (only 186 observations available for 2012 and 185 for 2013). SIZE is available to us on an annual basis from 2006 to 2012 (year-ends) from the OECD Economic Outlook database. The data on UPLIFT, SACR, and SCR are observations starting from end-2007 at annual frequency until and including end-2013.<sup>1</sup> The data are publicly available and have been collected from Moody's website.<sup>2</sup>

#### 3.03 Estimation results

#### 3.3.1 Comparison with theoretical predictions

We use a multinomial ordered logit model to perform the estimation of a model of the value of bank debt rating uplift, UPLIFT, consisting of the four explanatory variable described above (that is SACR, SCR, SIZE, and ASSETS); we run separate regressions for each of the seven years of our sample.

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<sup>&</sup>lt;sup>1</sup> The credit rating data for year T is matched against GDP data for year T-1, given that we wanted to use as much data as possible and as GDP data for any year T only becomes available with a delay, during year T+1.

<sup>&</sup>lt;sup>2</sup> See for more details Schich and Lindh (2012).

Consistent with the model implications discussed in Section 2.04, the relationship between the value of UPLIFT and the bank intrinsic strength holds empirically, regardless of the specific year. The estimated coefficient of bank intrinsic strength (SACR) is negative and always highly significant, regardless of the year considered: Weaker banks tend to benefit from greater UPLIFTS than do relatively stronger banks.

Also consistent with the conceptual considerations in Section 2.04, the relationship between the value of UPLIFT and the sovereign strength is positive, regardless of the year chosen. The relationship turns out to be insignificant however in the regressions for the years 2007 and 2008, but it is very highly significant for the years 2009 to 2013. The coefficient estimate is relatively stable within these three years, varying from 0.22 to 0.39 in value.

The relationship between the value of UPLIFT and the size, as measured by domestic GDP, of the sovereign where the banks are headquartered changes signs. It is negative from 2007 to 2009 and positive thereafter. It is significant and highly so from 2010 to 2013, where it is positive, consistent with the predictions of the discussion in Section 2.04. Bank size is highly significant in the expected direction throughout the whole sample.

Table 02 shows the results for the regression using data for 2013. All three variables are highly significant and have the expected signs. The variable UPLIFT is bigger for (1) a lower stand-alone credit rating of the bank, SACR, (2) a higher issuer credit rating of the respective sovereign, SCR, (3) a larger sovereign SIZE, and (4) a larger bank size, ASSETS. Appendix Table A.1 shows summary results for the other years.

Considering as dependent variable the changes over time rather than the levels of credit rating uplifts at one point in time, we find that the changes in bank and sovereign strength are highly significant in almost all specifications, while changes in the sovereign SIZE is significant only in some and changes in bank size in none of the specifications (Appendix Table A.2). Again, the effect of changes in bank strength dominate those in sovereign strength.

The results presented here are consistent with those obtained by previous empirical studies, which suggest that bank-specific factors matter as well as those related to the sovereign of the country in which the bank is located. Note in this context that the absolute size of the coefficient of the stand-alone rating SACR is larger than that of the coefficient of the sovereign rating SCR. This relationship is consistent with the relative magnitudes implied by the theoretical model for the local sensitivities with regard to bank and sovereign strength, as discussed in Section 2.04 and Appendix A.1.

Another way to look at this relationship is by computing the estimated marginal probability effects of increases in the values of explanatory variables.<sup>3</sup> Considering an increase by three notches in either one of these two explanatory variables, i.e. SACR or SCR, and the parameter estimates shown in Table 02, we find that changes in the former dominate those in the latter in most instances. The estimated sensitivities depend on the specific "starting points", however. For example, when considering a median strong bank (with a rating equivalent to a numerical equivalent of 11) and a weak sovereign (rating of 1), an increase in sovereign strength by three notches has a stronger effect than a similar increase of the banks' own strength. In the other cases considered here, the banks' own strength matters more than that of the sovereign, however (see Appendix Table A.3).

Table 02: Ordered logit regression for credit rating uplift at end-2013 (dependent variable is UPLIFT in						
numerical equivalents)						
Variable	Coefficient	Std. error	Probability			
Issuer bank stand-alone credit rating (SACR)	-0.52***	0.06	0.00			
Domestic sovereign credit rating (SCR)	0.34***	0.05	0.00			
Size of sovereign (SIZE)	0.20***	0.06	0.00			
Bank size (ASSETS)	0.70***	0.10	0.00			
Pseudo R-squared 0.50						

Notes: Observations for 185 large banks rated by Moody's at December 2013. \*\*\*,\*\*, and \* denote significance at the 1, 5, and 10 per cent level, respectively. The ratings categories ICR (all-in credit rating; not shown here, but used to calculate UPLIFT), SACR and SCR are transformed into numerical values (i.e. Aaa equal to 20, Aa1 equal to 19, etc.). UPLIFT is obtained by subtracting SACR from ICR. SIZE is measured by the logarithm of domestic GDP for the sovereign where the bank is headquartered and bank size, ASSETS, are measured by the logarithm of bank assets. See Estrella (1998) for information about the pseudo R-squared measure.

# 4.0 Concluding remarks

Based on a contingent claims model to value sovereign guarantees of bank debt, this paper demonstrates that the value of a guarantee of risky debt depends both on the characteristics of the borrower and the guarantor. For a

<sup>&</sup>lt;sup>3</sup> The authors are grateful to the discussant of the present paper at the IRMC 2014 for this suggestion.

given bank, the value of a government guarantee for its debt decreases with the bank's own creditworthiness and it increases with the creditworthiness and size of the sovereign perceived to provide the guarantee, as well as with the size of the bank. While both bank-specific and sovereign-specific factors determine the value of sovereign guarantees for bank debt, the banks' own strength matters more in the sense that the value uplift is locally more sensitive to the strength of the bank than to the strength of the guaranter.

These implications are consistent with the empirical findings from an analysis of a measure of implicit guarantees for (unsecured senior) bank debt for a sample of large banks worldwide for different points in time between 2007 and 2013. In particular, we find that this measure is higher for (1) a lower stand-alone credit rating of the bank, (2) a higher issuer credit rating of the respective sovereign, (3) larger sovereign size, and (4) larger bank size. The absolute size of the coefficient of the stand-alone rating is larger than that of the coefficient of the sovereign rating.

These results have implications for the assessment of the effects of bank regulatory reform. In particular, as part of such efforts, policy makers have clearly announced their intention to rein in the value of implicit bank debt guarantees. One difficulty with this approach is that not enough is known about the economic determinants of the value of implicit bank debt guarantees and how changes over time should be interpreted. The present paper sheds light on these issues and thus supports current policy efforts to limit the funding cost advantage of banks considered to benefit from an implicit bank debt guarantee.

They also have implications for the specific issue of pricing explicit emergency sovereign guarantees for bank debt to achieve or maintain financial stability. Such guarantees were provided by many governments in OECD countries between 2008 and 2010 and the European Financial Stability Facility (EFSF) in response to the financial crisis (see e.g. Grande, Levy, Panetta and Zaghini, 2011; Levy and Schich, 2010) and they might be provided by arrangements such as the European Stability Mechanism (ESM). In a situation where the guarantees are provided separately by each country, to avoid competitive distortions, the premiums for guarantees need to reflect both the strength of the bank and the sovereign, although the former should have somewhat more weight. In a situation where bank debt guarantees are provided jointly by several sovereigns, the allotment of premium income among the participating sovereigns should reflect each sovereign's creditworthiness. Stronger sovereigns should receive higher shares of premium incomes than weaker sovereigns even for identical amounts of committed or used guarantees.

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#### Appendix A.1. Relative Sensitivity of Uplift to Sovereign and Bank Risk

For initial sovereign and bank asset values  $(A_B, A_S)$ , the joint lognormal density function for asset values at time  $\tau$  is

$$f(x,y) = \frac{1}{2\pi\sigma_B\sigma_S\tau\sqrt{1-\rho^2}} \frac{1}{xy} \exp\left(-\frac{u(x)^2 + v(y)^2 - 2\rho u(x)v(y)}{2(1-\rho^2)}\right)$$
(A.1.1)

where  $u(x) = \left(\log(x/A_B) - \left(r - \frac{1}{2}\sigma_B^2\right)\tau\right) / \sigma_B \sqrt{\tau}$  and  $v(y) = \left(\log(y/A_S) - \left(r - \frac{1}{2}\sigma_S^2\right)\tau\right) / \sigma_S \sqrt{\tau}$ . The value of

stand-alone bank debt payable at time  $\tau$  is obtained from the marginal distribution  $f_B(x) = \int_0^\infty f(x, y) dy$  implied by (A.1) as

$$V_{B} = e^{-r} \left[ \int_{L_{B}}^{\infty} L_{B} f_{B}(x) dx + \int_{0}^{L_{B}} x f_{B}(x) dx \right].$$
(A.1.2)

For bank debt guaranteed by the sovereign and payable at time  $\tau$ , the value is obtained from the bivariate distribution (A.1) as

$$V_{G} = e^{-r} \left[ \int_{0}^{\infty} \int_{L_{B}}^{\infty} L_{B} f(x, y) dx dy + \int_{L_{B}+L_{S}-x}^{\infty} \int_{0}^{L_{B}} L_{B} f(x, y) dx dy + \int_{L_{S}}^{L_{B}+L_{S}-x} \int_{0}^{L_{B}} (x + y - L_{S}) f(x, y) dx dy + \int_{0}^{L_{S}} \int_{0}^{L_{B}} x f(x, y) dx dy \right].$$
(A.1.3)

So, uplift as a function of initial sovereign and bank assets is

$$U = U(A_B, A_S) = V_G - V_B.$$
 (A.1.4)

How sensitive is the uplift function to changes in sovereign assets, relative to its sensitivity to bank assets? One can calculate (see Estrella and Schich, 2011) that  $\partial U/\partial A_s > 0$  and  $\partial U/\partial A_B < 0$ . Thus, the relative sensitivity is given by the slope of an iso-uplift contour, defined as the locus of  $(A_B, A_S)$  such that  $U(A_B, A_S) = U_0$ , where  $U_0$  is a positive constant. This slope is given by

$$\frac{dA_B}{dA_S}\Big|_{U=U_0} = -\frac{\partial U/\partial A_S}{\partial U/\partial A_B} > 0.$$
(A.1.5)

**F**igures illustrating the relative sensitivity as given in Equation (A.5) for a range of values of  $(A_B, A_S)$  are presented in Section 2.04.

#### **Appendix A.2. Additional Tables and Figures**

Table A.1. Ordered logit regression for credit rating uplift for different years							
(dependent variable is UPLIFT in numerical equivalents)							
Year	2013	2012	2011	2010	2009	2008	2007
SACR	-0.53***	-0.56***	-0.51***	-0.85***	-0.94***	-0.96***	-1.00***
SCR	0.34***	0.33***	0.35***	0.39***	0.22***	0.08	0.10
SIZE	0.20***	0.22***	0.22***	0.15***	-0.03	-0.04	-0.07
ASSETS	0.70***	0.67***	0.80***	0.87***	0.98***	1.01***	1.07***
No. obs.	185	186	188	188	188	188	188
P-Rsq	0.50	0.52	0.51	0.65	0.70	0.62	0.62
Notes: Observatio	ns for large bar	nks rated by	Moody's in diff	erent years, fro	m 24 OECD c	countries. ***,**	, and * denote

significance at the 1, 5, and 10 per cent level, respectively. For other explanations please see Table 03 in the main text.

Table A.2. Ordered logit regression for changes in credit rating uplift from different years to 2013

(depen	ident variable is c	change in UPI	IFT in numer	ical equivaler	its)	
2013 versus	2012	2011	2010	2009	2008	2007
ΔSACR	-0.96***	-1.05***	-0.93***	-0.91***	-0.58***	-0.60***
ΔSCR	0.25	1.22***	0.79***	0.82***	0.55***	0.60***
SIZE	0.06	0.03	0.09	0.15**	0.13**	0.10*
ASSETS	0.24	-0.10	-0.13	-0.12	0.10	0.10
No. obs.	185	185	185	185	185	185
P-Rsq	0.28	0.47	0.51	0.62	0.46	0.55
Notes: Observations for large b	anks rated by Mo	ody's in differ	ent years, from	n 24 OECD co	untries. ***,**	, and * denote
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significance at the 1, 5, and 10 per cent level, respectively. For other explanations please see Table 03 in the main text.	

Table A.3. Marginal probability effects of credit rating increase by	three notches
(using parameter estimates shown in Table 02)	

		,		
	No uplift	Moderate uplift	Substantial uplift	
Weak bank, weak sovereign, SACR improves	13%	24%	-37%	
Weak bank, weak sovereign, SCR improves	-3%	-16%	18%	
Absolute difference in marginal effects	10%	9%	19%	
Weak bank, median sovereign, SACR improves	0%	1%	-1%	
Weak bank, median sovereign, SCR improves	0%	0%	0%	
Absolute difference in marginal effects	0%	1%	1%	
Median bank, weak sovereign, SACR improves	9%	-8%	-1%	
Median bank, weak sovereign, SCR improves	-15%	13%	2%	
Absolute difference in marginal effects	-6%	-5%	-1%	
Median bank, median sovereign, SACR improves	10%	27%	-37%	
Median bank, median sovereign, SCR improves	-2%	-14%	16%	
Absolute difference in marginal effects	8%	13%	21%	

Notes: Increase in probability of falling into one of three categories for a 3 notch increase in either bank or sovereign credit strength, keeping other factors constant. "No uplift" refers to a credit rating uplift of zero; "moderate uplift" refers to an uplift of 1 or 2 notches, and "substantial uplift" to at least 3 notches. Based on parameters estimates from ordered logit regression of UPLIFT on BSACR, SOVCR, SIZE and ASSETS as of 2013, as shown in Table 02. A "weak bank" and "weak sovereign" are assumed to have a SACR and SCR, respectively equal to one and a "median bank" and a "median sovereign" are assumed to have a SACR equal to 11 and a SCR equal to 17, respectively. The logarithm of GDP is kept constant at 32.4 and the logarithm of assets at 12.5.